

Calculation Policy

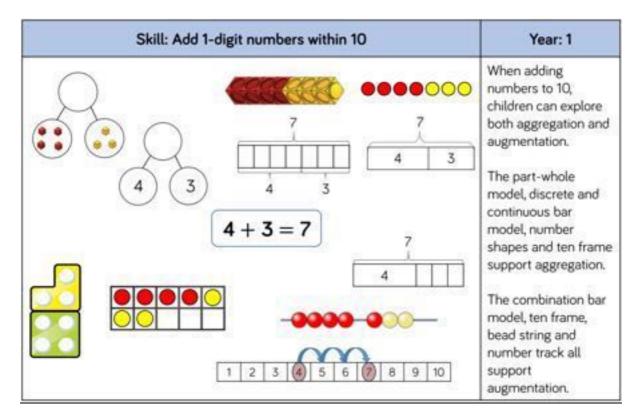
At Redfield Educate Together we believe that we should support all children to aspire to be mathematicians. We will help children to explore the connections between the different areas of maths and develop their mathematical thinking. We will develop the children's multidimensional fluency, ensuring that children have efficient, accurate and flexible methods for solving problems. Mathematical concepts will be explained using a variety of appropriate representations and models to aid understanding. We will ensure that a variety of procedural and conceptual maths is taught so that the children understand the "how" and the "why" of what they are doing. We hope to inspire a life-long love of mathematics and ensure the children have the skills to solve real-life, everyday problems as they grow older.

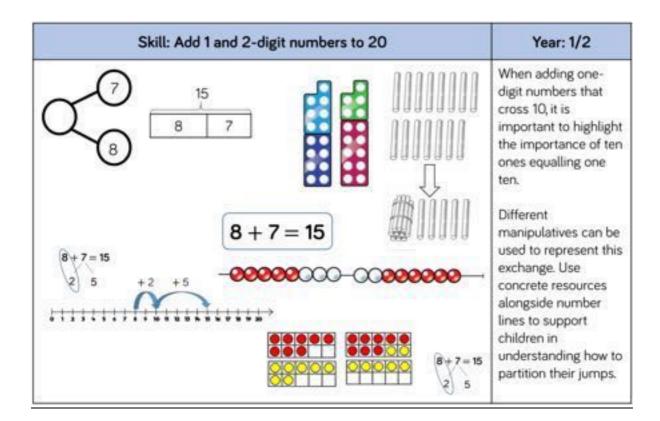
The Redfield Educate Together calculation policy contains the written procedures that will be taught within our school alongside practical resources. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

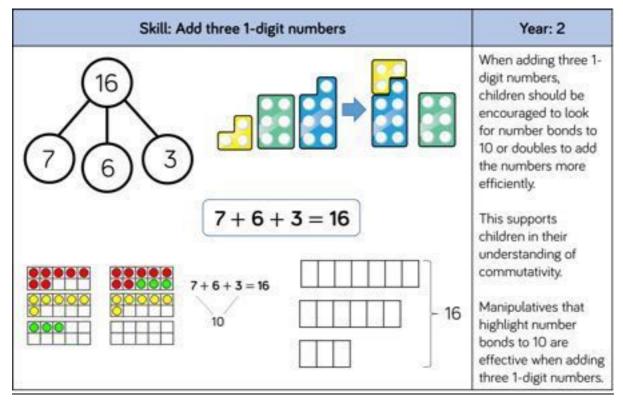
The document is broken down into addition, subtraction, multiplication and division. Each operation is then broken down into skills and each skill has a dedicated image showing representations and models that could be used to effectively teach that concept. There is an overview of the progression of skills linked to year groups to support cohesion across the school and a glossary of terms, representations and models that can support the teaching of different concepts.

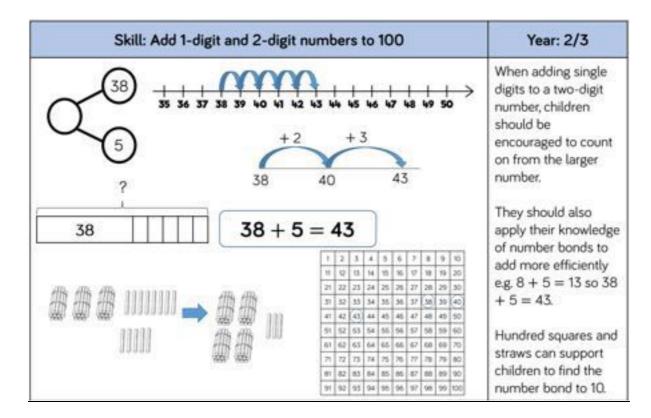
<u>Addition</u>

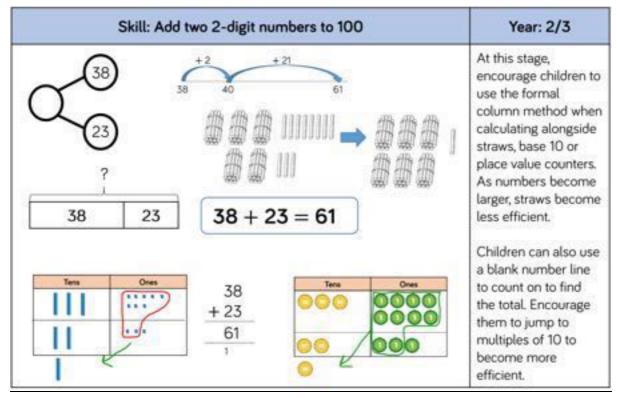
Skill	Year	Representation and models
Add two 1-digit numbers to 10	1	Part-whole model, Bar model, Number shapes, Ten
		frames (within 10), Bead strings (10), Number tracks
Add 1 and 2-digit numbers to	1	Part-whole model, Bar model, Number shapes, Ten
20		frames (within 20), Bead strings (20), Number tracks,
		Number lines (labelled), Straws
Add three 1-digit numbers	2	Part-whole model, Bar model, Number shapes, Ten
		frames (within 20),
Add 1 and 2-digit numbers to	2	Part-whole model, Bar model, Number lines (labelled),
100		Number lines (blank), Straws, Hundred square
Add two 2-digit numbers	2	Part-whole model, Bar model, Number lines (blank),
		Straws, Base 10, Place value counters
Add with up to 3-digits	<u>3</u>	Part-whole model, Bar model, Basse 10, Place value
		counters, Column addition
Add with up to 4-digits	4	Part-whole model, Bar model, Basse 10, Place value
		counters, Column addition
Add with more than 4-digits	5	Part-whole model, Bar model, Place value counters,
		Column addition
Add with up to 3 decimal places	5	Part-whole model, Bar model, Place value counters,
		Column addition

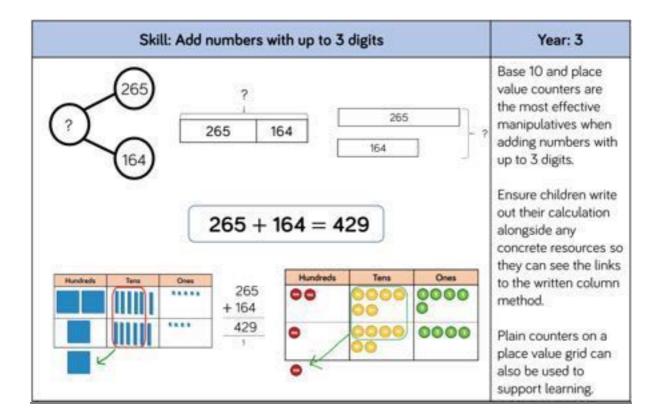


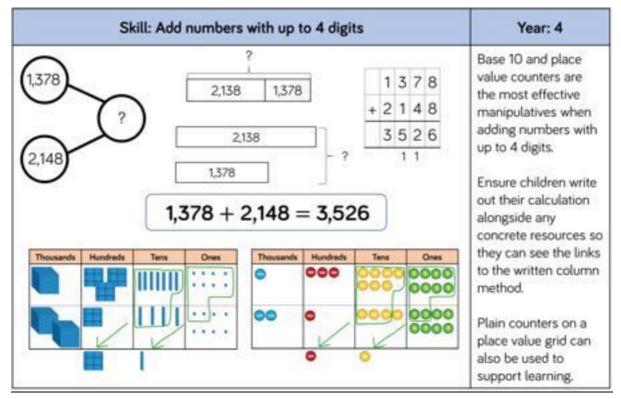


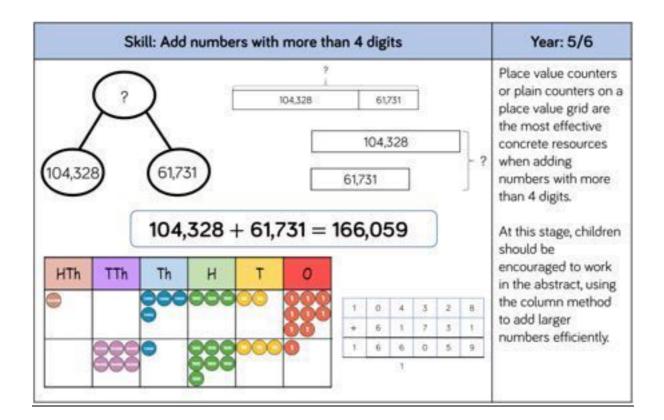


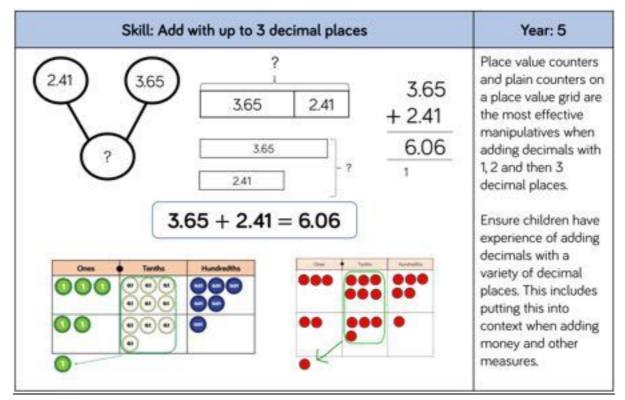






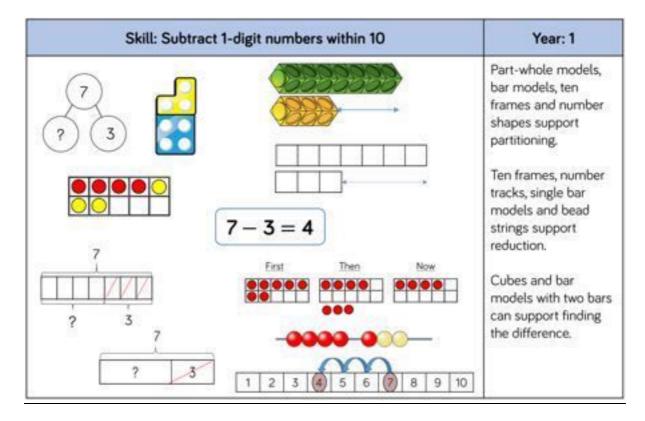


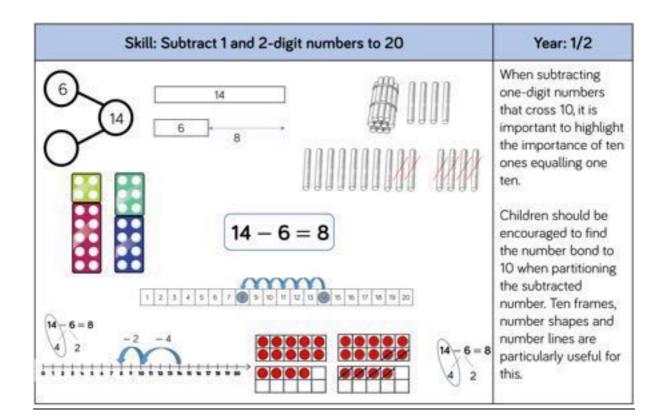


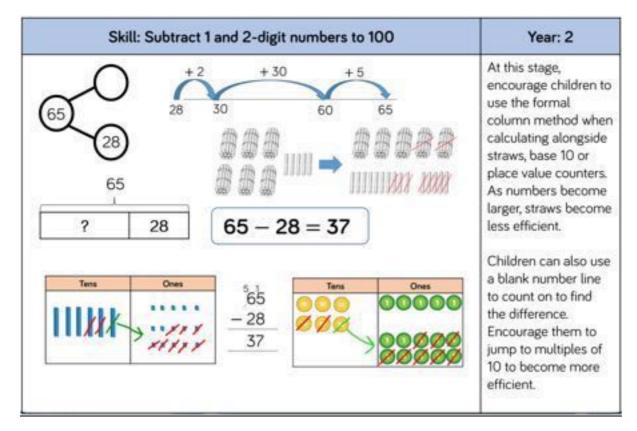


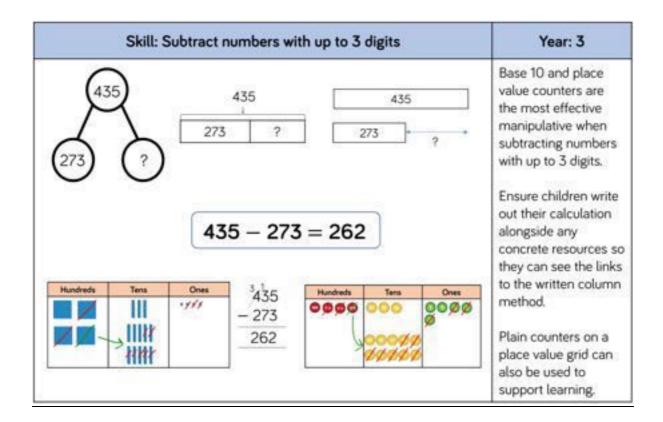
Subtraction

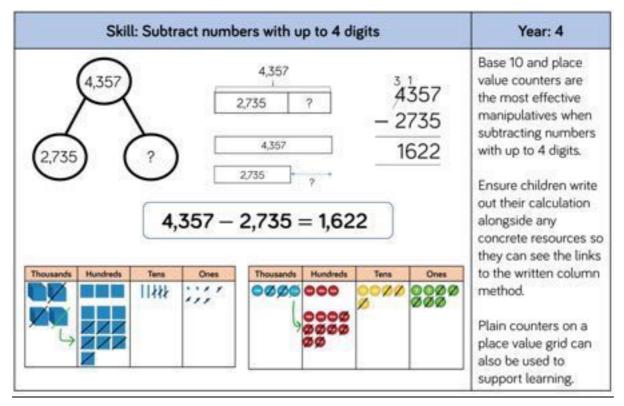
Skill	Year	Representation and models
Subtract 2 1-digit numbers to	1	Part-whole model, Bar model, Number shapes, Ten
10		frames (within 10), Bead strings (10) Number tracks
Subtract 2 1-digit numbers to	1	Part-whole model, Bar model, Number shapes, Ten
20		frames (within 20), Bead strings (20) Number tracks,
		Number lines (labelled), Straws
Subtract 2 1-digit numbers to	2	Part-whole model, Bar model, Number lines (labelled),
100		Number lines (blank), Straws, Hundred square
Subtract two 2-digit numbers	2	Part-whole model, Bar model, Number lines (blank)
		Straws, Base 10, Place value counters, Column addition
Subtract with up to 3-digits	3	Part-whole model, Bar Model, Base 10, Place value
		counters, Column addition
Subtract with up to 4-digits	4	Part-whole model, Bar Model, Base 10, Place value
		counters, Column addition
Subtract with more than 4-	5	Part-whole model, Bar Model, Place value counters,
digits		Column addition
Subtract with up to 3 decimal	5	Part-whole model, Bar Model, Place value counters,
places		Column addition

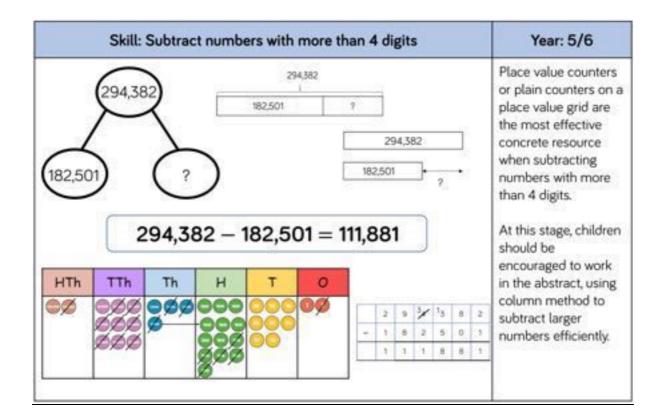


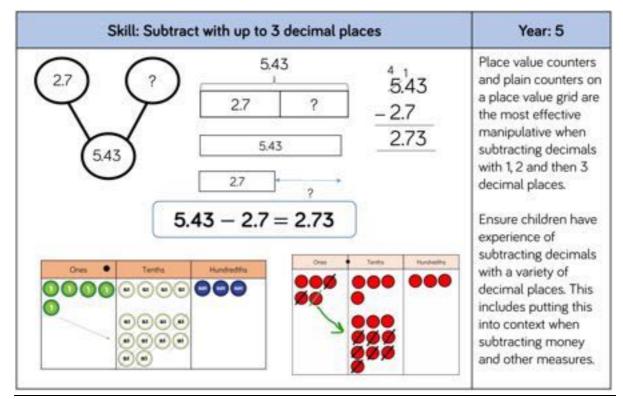






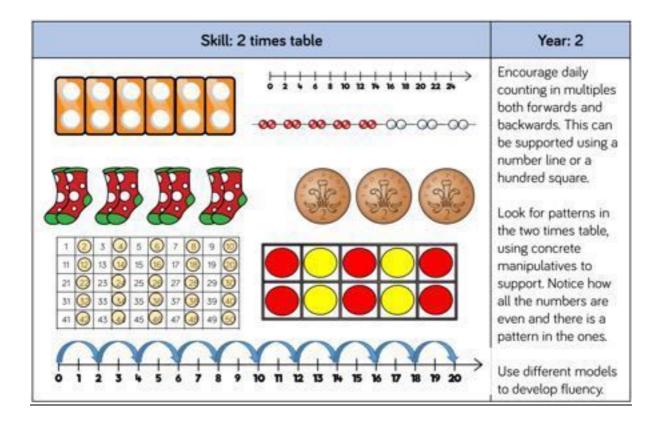






<u> Times Tables</u>

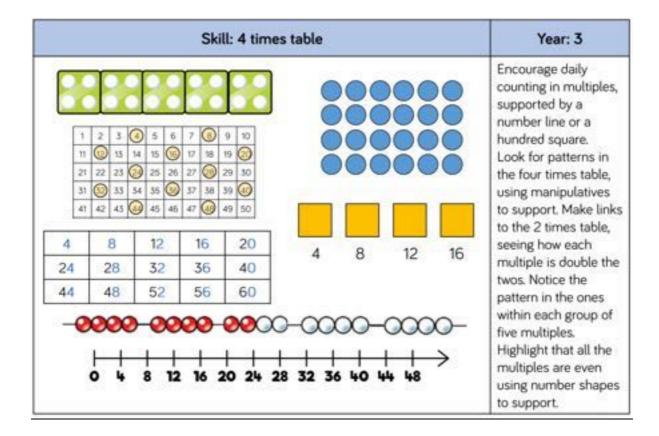
Skill	Year	Representation and models
Recall and use multiplication and division facts for the 2- times table	2	Bar model, Number shapes, Counters, Money, Ten frames, Bead strings, Number lines, Everyday objects
Recall and use multiplication and division facts for the 2- times table	2	Bar model, Number shapes, Counters, Money, Ten frames, Bead strings, Number lines, Everyday objects
Recall and use multiplication and division facts for the 10- times table	2	Hundred square, Number shapes, Counters, Money, Ten frames, Bead strings, Number lines, Base 10
Recall and use multiplication and division facts for the 3- times table	3	Hundred square, Number shapes, Counters, Bead strings, Number lines, Everyday objects
Recall and use multiplication and division facts for the 4- times table	3	Hundred square, Number shapes, Counters, Bead strings, Number lines, Everyday objects
Recall and use multiplication and division facts for the 8- times table	3	Hundred square, Number shapes, Bead strings, Number lines, Everyday objects
Recall and use multiplication and division facts for the 6- times table	4	Hundred square, Number shapes, Bead strings, Number lines, Everyday objects
Recall and use multiplication and division facts for the 7- times table	4	Hundred square, Number shapes, Bead strings, Number lines
Recall and use multiplication and division facts for the 9- times table	4	Hundred square, Number shapes, Bead strings, Number lines
Recall and use multiplication and division facts for the 11- times table	4	Hundred square, Base 10, Place value counters, Number lines
Recall and use multiplication and division facts for the 12- times table	4	Hundred square, Base 10, Place value counters, Number lines



								Skill: 5	times table	Year: 2
	1 9)(h s	e la	n Cel		m		r B Ceto		be supported using a number line or a hundred square. Look for patterns in
1	2	3	4	6	6	7	8	9 🕥		the five times table, using concrete
11	12	13	14	1	16	17	18	19 🛞		manipulatives to
21	22	23	24	0	26	27	28	29 🕑		support. Notice the
31	32	33	34	0	36	37	38	39 🕢		pattern in the ones a
41	42	43	44	0	46	47	48	49 🕢		well as highlighting
-	+	1 2	+ 3	+++++++++++++++++++++++++++++++++++++++	5	+ 6	+7	++	ю п 12 13 14 15 16 17 18 19 20	→ the odd, even, odd, even pattern.

Skill: 1	Year: 2										
		+20	***	+++++++++++++++++++++++++++++++++++++++		**** ***			~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	→	Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.
	1	2	3	4	5	6	7	8	9	\odot	Look for patterns in
	11	12	13	14	15	16	17	18	19	0	the ten times table,
	21	22	23	24	25	26	27	28	29	9	using concrete
	31	32	33	34	35	36	37	38	39	0	manipulatives to
	41	42	43	44	45	46	47	48	49	0	support. Notice the
	51	52	53	54	55	56	57	58	59	0	pattern in the digits-
	61	62	63	64	65	66	67	68	69	0	the ones are always 0,
	71	72	73	74	75	76	77	78	79	8	and the tens increase
	81	82	83	84	85	86	87	88	89	\otimes	by 1 ten each time.
	91	92	95	94	95	96	97	98	99	0	

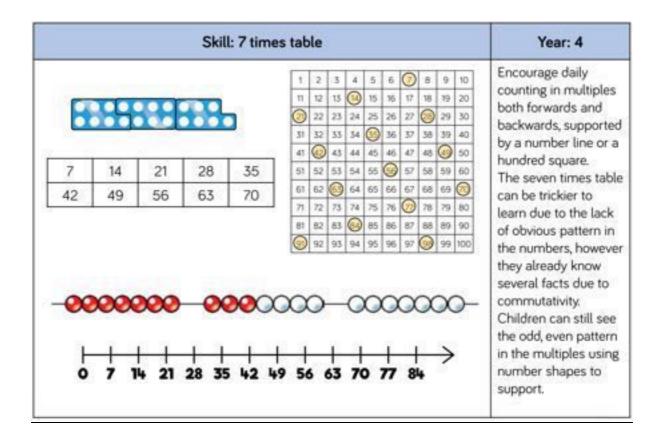
						1	Skill:	able	Year: 3
1 11	32		34	 6 16 26 346	37	8 00 28 38	 10 19 20 29 39 40 		Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the three times table,
41	42	43 Ho		+6	47		49 50 10	6 9 12 	using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.



Skill: 8 times table														Year: 3	
					3	2	3	4	5	6	7	0	9	10	Encourage daily
		-			11	12	-13	14	15	0	17	18	19	20	counting in multiples,
				•••	.21	22	23	0	25	26	27	28	29	30	supported by a number line or a
0	-		0	0	31	0	35	34	35	36	37	38	39	6	hundred square.
()	.(2 .	()	()	-41	42	43	44	45	46	47	۲	49	50	Look for patterns in
25	2 Spa	33	25	Sold State	51	52	53	54	55	9	57	58	59	60	the eight times table,
0	10	1.	24	70	61	62	63	6	65	66	67	68	69	70	using manipulatives
0	16	2	24	32	71	0	73	74	75	76	77	78	79	\odot	to support. Make links
			1 -		81	82	83	84	85	86	87	88	89	90	to the 4 times table,
8	16	24	32	40	91	92	93	94	95	96	97	98	99	100	seeing how each
48	56	64	72	80											multiple is double the
-0		2000 16 24	<mark>⊘ - ⊘(</mark> + + + • 32 +(2000 9 48 50	00 + 5 64	7	2	+	×	8 9	+				fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 6 times table														Year: 4	
00					1	2	3	4	5	(6)	7	8	9	10	Encourage daily
00					11	1	13	14	15	16	17	1	19	20	counting in multiples,
10	18	1.952	1003	Jul.	21	22	23	0		26	27	28	29	0	supported by a number line or a
H			R R		31	32	33	34	35	0	37	38	39	40	hundred square.
					41	0	43	44	45	46	47	0	49	50	Look for patterns in
					51	52	53	0	55	56	57	58	59	\odot	the six times table,
6	12	18	24	30	61	62	63			66	67	68		70	using manipulatives
36	42	48	54	60	21	72	73	74	75	76	77	78	.79		to support. Make link
					.81	82	83	84	85	86	87	88	89	90	to the 3 times table,
66	72	78	84	90	91	92	93	94	95	96	97	98	99	100	seeing how each
-		+++	+	0000 		8 !	+	+ 60			+ 72)		multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 9 times table														Year: 4	
_			_	-	t	2	3	4	5	6	7.	8	0	10	Encourage daily
++	н			••••	11	12	13	14	15	16	17	•	19	20	counting in multiple
				~~~	21	22	23	24	25	26	0	28	29	30	both forwards and backwards. This can
					31	32	33	34	35	0	37	38	39	40	be supported using
					-41	42	43	44	6	46	47	48	49	50	number line or a
9	18	27	36	45	51	52	53	0	55	56	57	58	59	60	hundred square.
54	63	72	81	90	61	62	0	64	65	66	67	68	69	70	Look for patterns in
-					71	0	73	74	75	76	77	78	79	80	the nine times table
					(3)	82	83	84	85	86	87	88	89	0	using concrete
					91	92	93	94	95	96	97	98	9	100	manipulatives to
-00	0000	20000	- <b>o</b>	00000	000	>-	-0		0	00	0	200	0	_	support. Notice the pattern in the tens and ones using the hundred square to

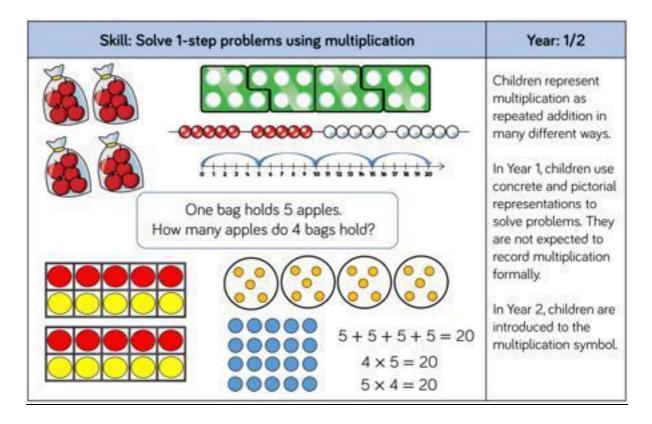


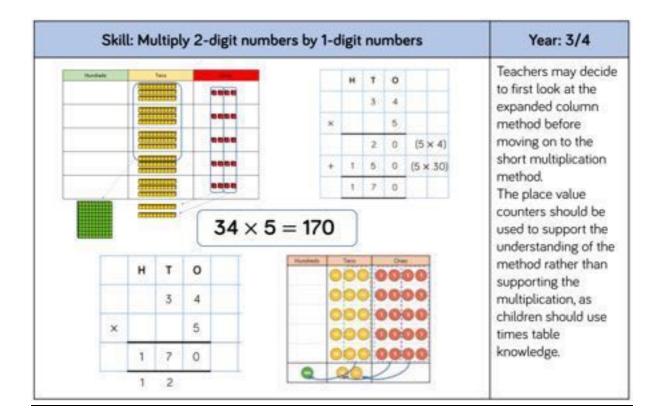
Year: 4									le	es tab	: 11 tim	Skill				
Encourage daily	10	9	8	7	6	5	4	3	2	1	66	55	44	33	22	
counting in multiple both forwards and	20	19	18	17	16	15	14	13	12	0		10.00	110	10000	-	-
backwards. This ca	30	29	28	27	26	25	1.1.1.	23	0	21	132	121	110	99	88	7 8
be supported using	40	39	38	37	36	35	-	•	32	35	-	-	0	0		-
number line or a	50	49	48	47	11.2	1.0	۲	43	42	41		C		$\odot$		
hundred square.	60	59	58	57	56	0	54	53	52	51		C	0			
nonoreo aquare	70	69	68	67	$\Theta$	65	64	63	62	61		2		$\overline{}$		
Look for patterns in	80	79	78	0	76	75	74	73	72	71		0				
the eleven times			⊛	87	86	85	84	83	82	81						
table, using concret	100	9	98	97	96	95	94	93	92	91						
manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the patter after crossing 100			52	E	121		T	+ 99	8	77 8	5 66	44 5	33	22	+ 11	- + o

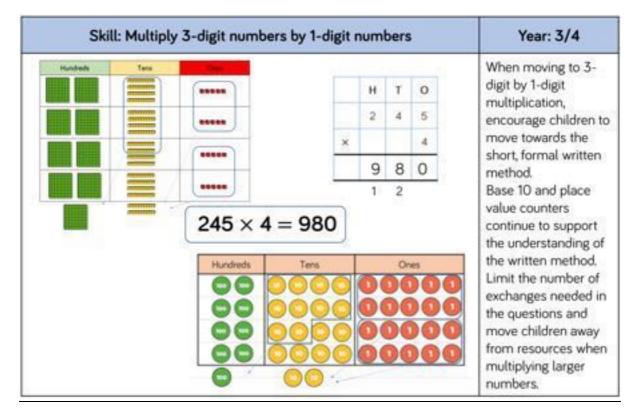
				Skil	: 12 times	tab	le									Year: 4
-					1	1	2	3	4	5	6	7	в	9	10	Encourage daily
12	24	36	48	60		11	1	13	14	15	16	17	18	19	20	counting in multiples
72	84	96	108	120	6 - 1	21	22	23	0	-	26	27	28	29	30	supported by a number line or a
170	1000					31	32	33	34	_	0	37	38	39	40	hundred square.
132	144				1	41	42	43	44	45	46	47	۲	49	50	Look for patterns in
0		0			000	51	52	53	54	55	55	57	58	59	0	the 12 times table,
		$\bigcirc$				61	62	63	64	65	66	67	68	69	70	using manipulatives
			1 1		000	71	0	73	1261	75	76	77	78	79	80	to support. Make link
		~	~~			81	82	83	0	85	86	87	88	89	90	to the 6 times table,
						91	92	93	94	96	0	97	98	99	100	seeing how each
	⊢ 0 1	2 24	+ 36	48	50 72 84	9		+104	8 12	20	+	2 74				multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

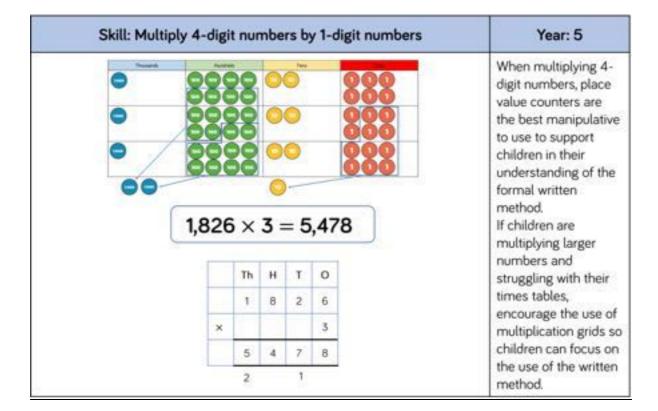
### **Multiplication**

Skill	Year	Representations and models
Solve one-step problems with	1/2	Bar model, Number shapes, Counters, Ten frame, Bead
multiplication		strings, Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters, Base 10, Short written method, Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place value counters, Base 10, Short written method
Multiply 4-digit by 1-digit numbers	5	Place value counters, Short written method,
Multiply 2-digit by 2-digit numbers	5	Place value counters, Base 10, Short written method, Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters, Short written method, Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method

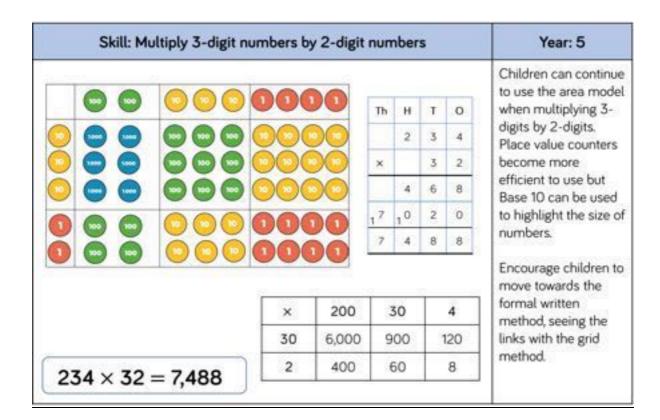








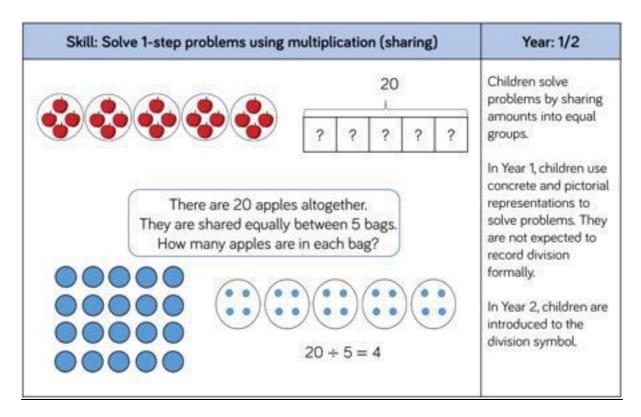
		Skill:	Year: 5									
30				2		0						When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a
	8			88					н	т	0	rectangle by finding the space covered by
					×	20	2			2	2	the Base 10.
1					30	600	60	×		3	1	The grid method matches the area
-	-				1	20	2			2	2	model as an initial
									6	6	0	written method before moving on to
	2	2 × 31	= 682	2	]				6	8	2	the formal written multiplication method.

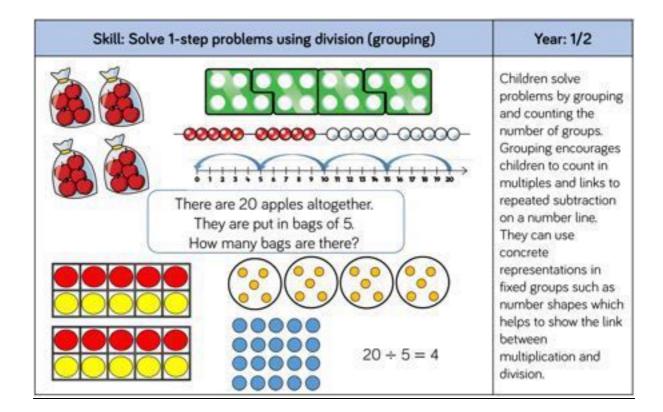


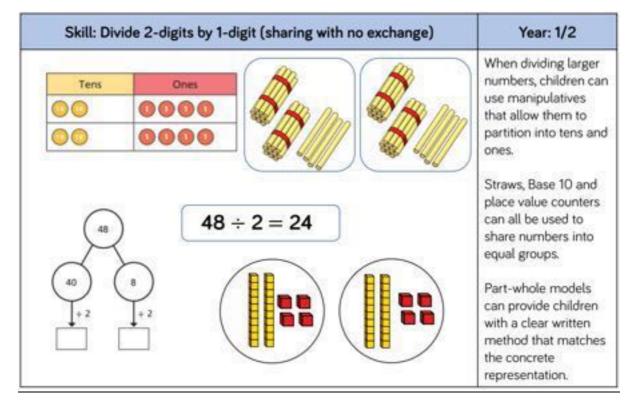
Skill: Multi	ply 4-di	Year: 5/6				
	TTh	Th	н	т	0	When multiplying 4- digits by 2-digits, children should be
		2	7	3	9	confident in the written method.
	×			2	8	If they are still struggling with times
	2	1	9 3	71	2	tables, provide multiplication grids to support when they
	15	4	, 7	8	0	are focusing on the use of the method.
	7	6	6	9	2	Consider where
			1			exchanged digits are placed and make
2,739 × 28 =	= 76,6	592				sure this is consistent

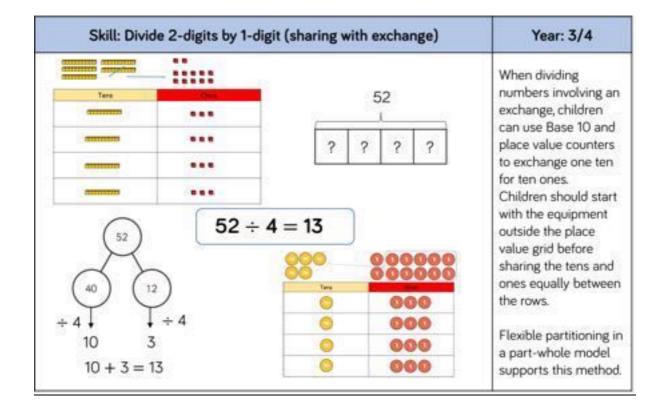
#### **Division**

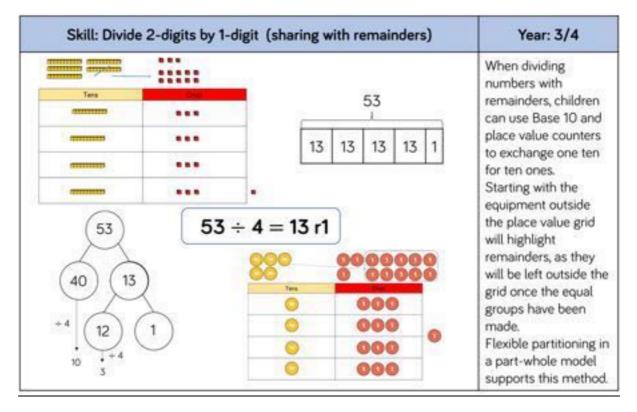
Skill	Year	Representation and model
Solve one-step problems with	1/2	Bar model, Real life objects, Arrays, Counters
division (sharing)		
Solve one-step problems with	1/2	Real life objects, Number shapes, Bead strings, Ten
division (grouping)		frames, Number lines, Arrays, Counters
Divide 2-digits by 1-digit (no	3	Straws, Base 10, Bar model, Place value counters, Part-
exchange sharing)		whole model
Divide 2-digits by 1-digit	3	Straws, Base 10, Bar model, Place value counters, Part-
(sharing with exchange)		whole model
Divide 2-digits by 1-digit	3/4	Straws, Base 10, Bar model, Place value counters, Part-
(sharing with remainders)		whole model
Divide 2-digits by 1-digit	4/5	Place value counters, Counters, Place value grid, Written
(grouping)		short division
Divide 3-digits by 1-digit	4	Base 10, Bar model, Place value counters, Part-whole
(sharing with exchange)		model
Divide 3-digits by 1-digit	4/5	Place value counters, Counters, Place value grid, Written
(grouping)		short division
Divide 4-digits by 1-digit	5	Place value counters, Counters, Place value grid, Written
(grouping)		short division
Divide multi-digits by 2-digits	6	Written short division, List of multiples
(short division)		
Divide multi-digits by 2-digits	6	Written long division, List of multiples
(long-division)		

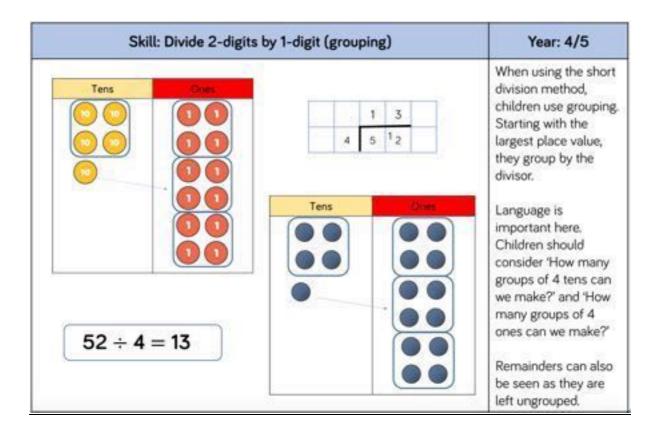




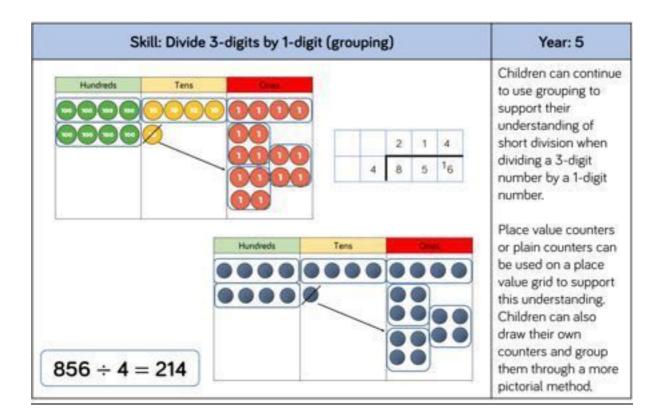


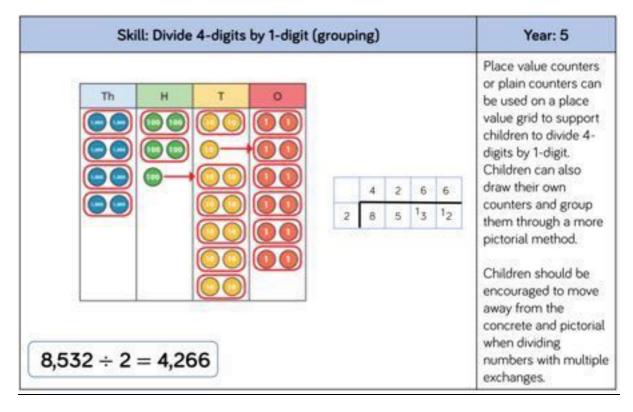






Skill: [	Year: 4			
$844 \div 4 = 122$ $844$ $? ? ? ? ?$ $844 \div 4 = 122$				Children can continue to use place value counters to share 3- digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds tens and ones equally between the rows.
(856)	Hund	reds Terrs		This method can also
~~			0000	help to highlight remainders.
	(16)		0000	Flexible partitioning in
(800) (40) (				T VERTICIA POIL TURNET IN US IT
	1+4 O		0000	a part-whole model





Year: 6	)	vision)	ort div	its (sh	y 2-dig	digits b	e multi	Divid	Skill:	
When children begin to divide up to 4- digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective Children can write out multiples to support	6	2 = 3	÷ 12	432	0 3 6 12 4 4 3 7 2					
their calculations with larger remainders.	9	8	4	0		_				
Children will also solve problems with remainders where the	¹³ 5	135	73	7	15	7,335 ÷ 15 = 489				
remanioers where u				j21;	و آن		_		_	

1 5 3 7 2 2×15=30 le												Year: 6				
										2	4	1	1	2	1 × 15 = 15	When a remainder is
							1	5	3	7	2				2 × 15 = 30	left at the end of a
372 ÷ 15 = 24 r12							-	3	0	0					calculation, children can either leave it as	
5/2	÷	15	-	- 4	4	r12				7	2				4 × 15 = 60	remainder or convert
								-		6	0				5 × 15 = 75 10 × 15 = 150	it to a fraction.
										1	2		-		10 × 10 = 100	This will depend on the context of the
																question.
				2	4	4										
	1	5	3	7	2	- 9										Children can also answer questions
		-	3	0	0			-	70			5	_	2	4	where the quotient
				7	2			0	12			5	=	24	4 <u>4</u> 5	needs to be rounded
		-	-	6	0											according to the context.
				1	2											CONTRACT.



#### <u>Glossary of terms</u>

Addend- A number to be added to another.

Aggregation- combining two or more quantities or measures to find a total.

**Array-** An ordered collection of counter, cubes or other items in rows or columns.

Augmentation- increasing a quantity or measure by another quantity.

**Commutative-** numbers can be added in any order.

**Complement-** in addition, a number and its complement make a total e.g 300 is the complement to 700 to make 1,000.

**Difference-** the numerical difference between two numbers is found by comparing the quantity in each group.

**Dividend-** in division, the number that is divided.

**Divisor-** in division, the number by which another is divided.

**Exchange-** Change a number or expression.

Factor- A number that multiplies with another to make a product.

Multiplicand- In multiplication, a number to be multiplied by another.

**Minuend-** A quantity or number from which another is subtracted.

**Partitioning-** Splitting a number into its component parts.

**Product-** The result of multiplying one number by another.

**Quotient-** The result of division.

**Reduction-** Subtraction as take away.



**Remainder-** The amount left over after a division when the divisor is not a factor of the dividend.

**Scaling-** Enlarging or reducing a number by a given amount, called the scale factor.

**Subtrahend-** A number to be subtracted from another.

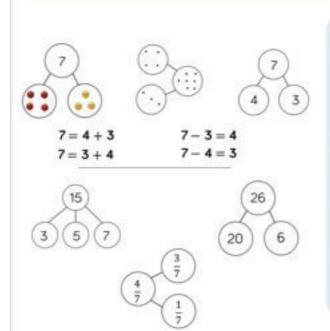
**Subitise-** Instantly recognise the number of objects in a small group without needing to count.

**Sum-** The result of an addition.



### Glossary of representations and models (addition and subtraction).

# Part-Whole Model



# Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

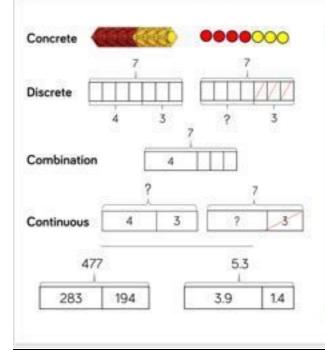
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

## Bar Model (single)



# Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

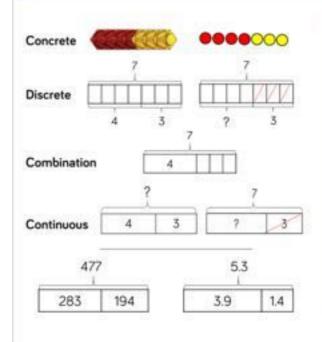
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

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In KS2, children can use bar models to represent larger numbers, decimals and fractions.

# Bar Model (multiple)

 Discrete

 Image: Discrete

# Benefits

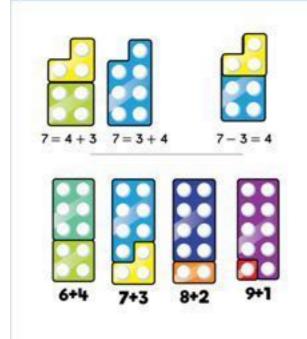
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

### **Number Shapes**



## Benefits

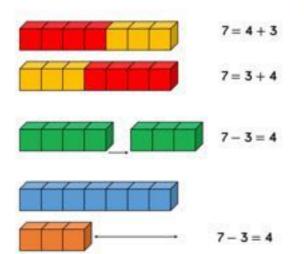
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

### Cubes



# Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

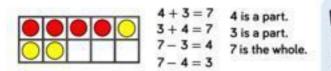
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

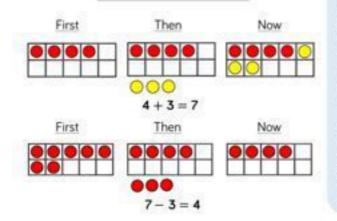
When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

# Ten Frames (within 10)





## Benefits

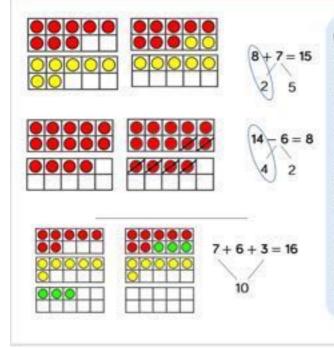
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

# Ten Frames (within 20)



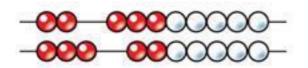
# Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

## **Bead Strings**







# Benefits

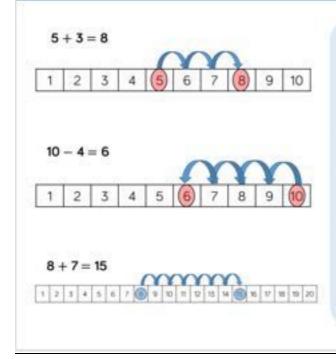
Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

### Number Tracks



# Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

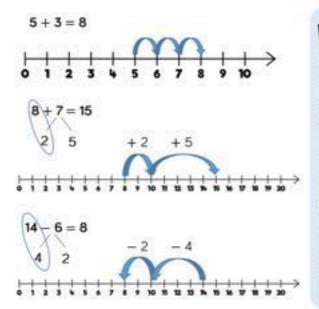
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

# Number Lines (labelled)



# Benefits

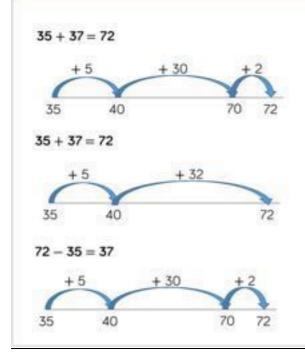
Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

# Number Lines (blank)



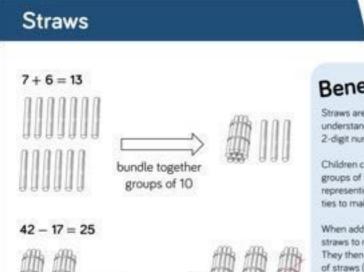
# Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.



# Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

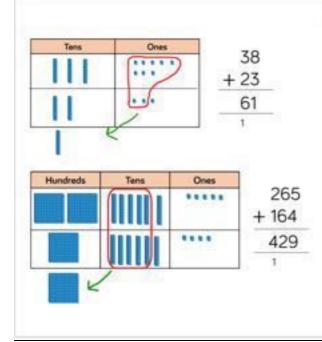
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

# Base 10/Dienes (addition)

unbundle group

of 10 straws



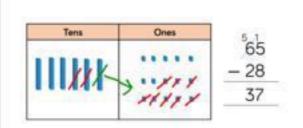
# Benefits

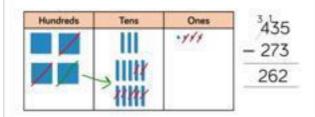
Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children. How many ones are there altogether? Can we make an exchange? (Yes or No) How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

### Base 10/Dienes (subtraction)





### Benefits

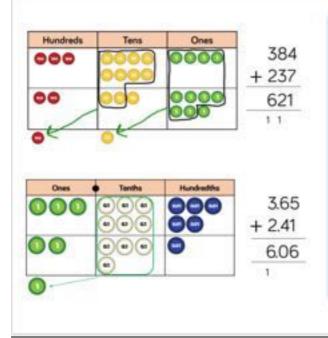
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough

ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

# Place Value Counters (addition)



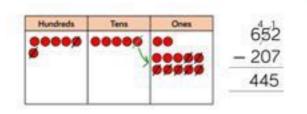
# Benefits

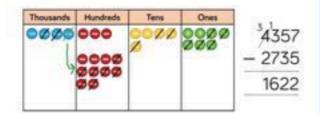
Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

## Place Value Counters (Subtraction)





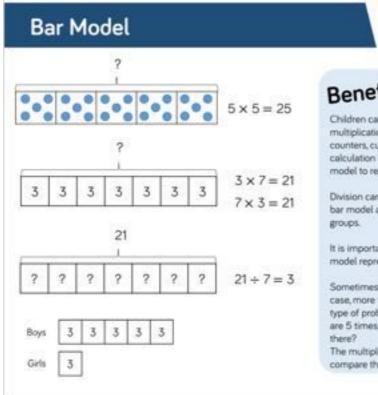
# Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

### Glossary of representations and models (multiplication and division).



# Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are

The multiple bar model provides an opportunity to compare the groups.

### Number Shapes







18 ÷ 3 = 6

### Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd  $\times$  odd = even, odd  $\times$  even = odd, even  $\times$  even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

### Bead Strings

-000	9-000-0	<mark>∞-∞</mark> 00-∞∞-	•
	5 × 3 = 15 3 × 5 = 15	15 ÷ 3 = 5	
-00	000-000	-00000-	
	$5 \times 3 = 15$ $3 \times 5 = 15$	15 ÷ 5 = 3	
-0000	0000 00	00-0000-0000	1
	1 5 - 20		

# Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

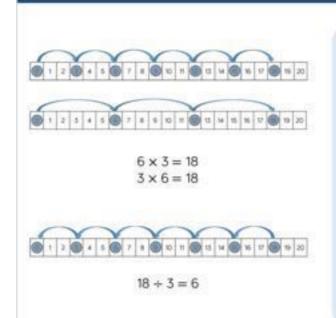
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

 $4 \times 5 = 20$   $5 \times 4 = 20$  $20 \div 4 = 5$ 

### **Number Tracks**



## Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

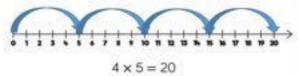
When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

## Number Lines (labelled)





 $5 \times 4 = 20$ 



 $20 \div 4 = 5$ 

# Benefits

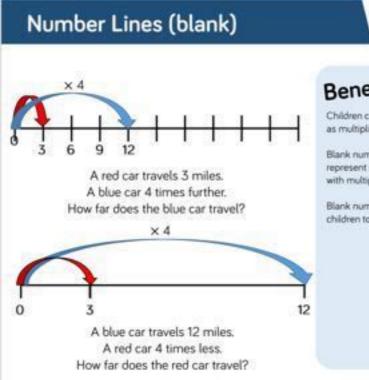
Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



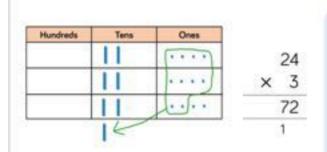
# Benefits

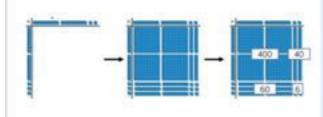
Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

### Base 10/Dienes (multiplication)





# Benefits

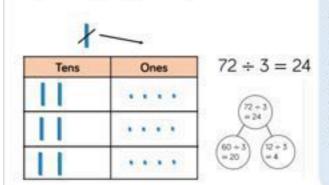
Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

### Base 10/Dienes (division)





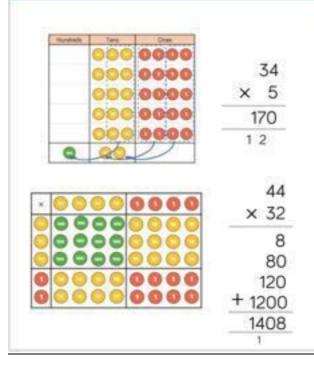
## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the partwhole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

# Place Value Counters (multiplication)



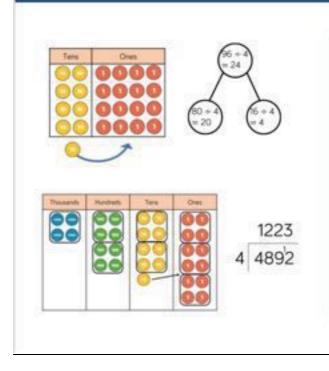
# Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2digit numbers by 2-digit numbers.

# Place Value Counters (division)



# Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.